Design Options of the Versatile Oversampling Two-Channel SBC-FDFMUX Filter Bank

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Abstract — Modern FDM communication systems demand highly efficient digital processors, which flexibly allow for allocating different bandwidths to different users [6]. Hence, for the processing of the incoming signal, two processor tasks must be distinguished: i) Mere demultiplexing of an FDM-signal and ii) demultiplexing of a single wideband signal with subsequent perfect reconstruction. This paper investigates and compares design options of the novel SBC-FDFMUX filter bank that efficiently combines both of the above tasks.

1 INTRODUCTION

Digital processors applied in modern FDM communication systems have to flexibly allow for allocating different channel bandwidths to different users. To obtain straightforward and manageable systems, in certain communication satellites using on-board switching for variable channel to beam allocation [6], the processor has to channelise the incoming FDM-signal always to a granularity level before re-multiplexing. Hence, the processor must efficiently realise two opposing tasks: i) Mere demultiplexing of the incoming FDM-signal (FDMUX) and, eventually, the re-multiplexing of the same or other independent granular signals (FMUX) calling for an FDFMUX filter bank and ii) demultiplexing of a single wideband signal with subsequent (almost) perfect reconstruction (PR) using an SBC filter bank. Both tasks are based on the efficient complex-modulated filter bank approach [3, 5] where, in the first case only, the application of $M$-th band filters further improves efficiency [2, 3].

In [1] a novel oversampling SBC-FDFMUX filter bank for two channels was proposed, efficiently combining both mutually exclusive processor functionalities. It is derived from the FDMUX Universal Directional Filter Cell (UNDIFICE) [2, 3] designed for a decimation factor of $M = 2$ and $I = 4$ channels, from which only two are used: This 4-channel UNDIFICE is split into two 2-channel FDMUX filter banks by assigning the even-numbered (odd-numbered) channels, $l = 0, 2$ ($l = 1, 3$), to a first (second) FDMUX filter bank. Subsequently, for instance, the first filter bank is complemented with a succeeding 2-channel FMUX filter bank to form an SBC filter bank [3], as depicted in figure 1. The Standard QMF (SQMF) and the Conjugated QMF (CQMF) approaches lend themselves to the design of an almost PR (NPR) or a PR SBC filter bank [3, 5], respectively. The suitability of these design approaches to the SBC-FDFMUX filter bank is investigated, and their features are discussed especially with respect to efficiency.

![Figure 1: The SBC-FDFMUX filter bank.](image)

In Section 2, the SBC-FDFMUX filter bank [1] is briefly recalled. Next, the SQMF and CQMF approaches are applied to the design of the SBC filter bank according to figure 1 subject to the requirement that $Y_0(z_0)$ and $Y_1(z_0)$ represent the complex-valued FDMUX output signals. Finally, the computational burdens of both approaches are compared with each other and with that of a pure FDMUX filter bank without SBC capability according to [2].

2 THE SBC-FDFMUX FILTER BANK

In the following the $z$-domain matrix description of the SBC-FDFMUX is recalled using alias component matrices [1, 3, 5]. Finally, the transition to polyphase matrices yields efficient realisations.

In the scope of this paper, an $R \times C$ matrix $M$ with $R$ rows and $C$ columns is denoted by using $M = [M(r, c)]_{r=1, \ldots, R; c=1, \ldots, C}$, where $M(r, c)$ is the element of $M$ in the $(r+1)th$ row and the $(c+1)th$ column. For a $Q \times Q$ diagonal matrix $D$ the diagonal elements are denoted by $D(q, q) = D(q)$ and $D = \text{diag}\{D(q)\}_{q=0,\ldots,Q-1}$.
2.1 The FDMUX Filter Bank

The centre frequencies of the channel filter transfer functions of the 4-channel UNDIFICE are allocated on the equidistant frequency grid:

\[ \Omega_{q}^{(i)} = 2\pi \frac{f_{l}}{f_{s}} l \frac{2\pi}{4} + \Delta \Omega_{q}^{(i)}, \]

where \( l \in \{0, 1, 2, 3\} \) is the channel index, \( f_{l} \) the input sampling frequency, and the frequency offset is set to \( \Delta \Omega_{q}^{(i)} = 0 \), subsequently. All filter functions

\[ H_{p}(z_{i}) = H_{p}(z_{i}W_{K}^{4}) \]

are derived from the real prototype filter \( H_{p}(z_{i}) \) by complex modulation (cf. figure 2) applying the frequency shifts according to (1), where \( W_{K} = e^{-j\frac{2\pi}{K}} \) with \( K = 4 \). Due to \( \Delta \Omega_{q}^{(i)} = 0 \), the even filters \( (l = 0, 2) \) are real-valued, whereas the odd filters \( (l = 1, 3) \) are complex-valued (indicated by underlining). Note that the FDMUX represents the analysis part of the SBC filter bank.

\[ \begin{bmatrix} \ddot{\bar{Y}}_{2}\left(z_{i}^{2}\right) \\ \dddot{\bar{Y}}_{2}\left(z_{i}^{2}\right) \\ \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \ddot{H}_{2}^{(ac)}(z_{i}) \end{bmatrix}^{T} \cdot \begin{bmatrix} \dddot{X}_{2}^{(ac)}(z_{i}) \end{bmatrix}, \]

Figure 2: The real prototype filter and the four potential filter functions of the FDMUX.

2.1.1 Alias Component Matrix Description

As for the SBC-FDMUX filter bank, each maximally decimating 2-channel FDMUX filter bank \((I = M = 2)\) is generally described by [1, 3, 5]

\[ \ddot{Y}_{2}\left(z_{i}^{2}\right) = \frac{1}{2} \begin{bmatrix} \ddot{H}_{2}^{(ac)}(z_{i}) \end{bmatrix}^{T} \cdot \dddot{X}_{2}^{(ac)}(z_{i}), \]

where the quantities \( \bullet \) are replaced with \( \hat{\bullet} \) (\( \hat{\bullet} \)) to represent the even-numbered (odd-numbered) FDMUX channels. Hence, the \( I \times 1 = 2 \times 1 \) vectors \( \ddot{Y}_{2}\left(z_{i}^{2}\right) = \left[ \dddot{Y}_{0}\left(z_{i}^{2}\right) \quad \dddot{Y}_{1}\left(z_{i}^{2}\right) \right]^{T} \) and \( \dddot{Y}_{2}\left(z_{i}^{2}\right) = \left[ \dddot{Y}_{1}\left(z_{i}^{2}\right) \quad \dddot{Y}_{2}\left(z_{i}^{2}\right) \right]^{T} \) comprise the respective two output channels. Similarly, the \( M \times I = 2 \times 2 \) FDMUX alias component matrices are determined by

\[ \dddot{H}_{2}^{(ac)}(z_{i}) = \left[ H_{i=2c}(z_{i}W_{K}^{4}) \right]_{k=0,1; \ c=0,1}, \]

\[ \dddot{H}_{2}^{(ac)}(z_{i}) = \left[ H_{i=2c+1}(z_{i}W_{K}^{4}) \right]_{k=0,1; \ c=0,1}, \]

containing the respective channel filter functions and their alias components. Finally, the \( M \times 1 = 2 \times 1 \) vector \( \dddot{X}_{2}^{(ac)}(z_{i}) = \left[ \dddot{X}_{2}\left(z_{i}W_{K}^{4}\right) \right]_{k=0,1; \ c=0,1} \) comprises the alias components of the input signal.

2.1.2 Efficient Polyphase Realisation [1]

The uniformness of the UNDIFICE stated by (1) and (2), respectively, and shared by the SBC-FDMUX filter bank, allows for an efficient realisation based on complex-modulated polyphase filter banks: The prototype filter is decomposed in its 1 type-1 (p1) polyphase components [3, 5]

\[ H_{p}(z_{i}) = \sum_{p=0}^{I-I} z_{i}^{-p}H_{p,1}^{(p1)}(z_{i}^{2}), \]

and the channel filter functions, given by (2), are obtained by applying the IFFT algorithm [3, 5]. The downsamplers at the system output are shifted to the system input using the noble identities leading to a higher computational efficiency [3, 5].

Following this approach, the alias component matrices (4) and (5) are decomposed in [1]:

\[ \dddot{H}_{2}^{(ac)}(z_{i})^{T} = W_{2}E_{2}^{(p1)}(z_{i}^{2})D_{2}^{(p1)}(z_{i})W_{2}^{*}, \]

\[ \dddot{H}_{2}^{(ac)}(z_{i})^{T} = W_{2}D_{2}^{(p1)}(z_{i}) \left( e^{-j\frac{2\pi}{4}} \right) E_{2}^{(p1)}(-z_{i}^{2}) \cdot D_{2}^{(p1)}(z_{i})W_{2}^{*}. \]

Here, \( W_{2} = \left[ W_{2}^{-p} \right]_{p=0,1} \) is the conjugated 2 \times 2 DFT matrix and \( D_{2}^{(p1)}(z_{i}) = \text{diag}\{z_{i}^{-q}\}_{q=0,1} \) is a 2 \times 2 delay matrix. The 2 \times 2 diagonal polyphase matrix \( E_{2}^{(p1)}(z_{i}^{2}) = \text{diag}\{H_{p,1}(z_{i}^{2})\}_{p=0,1} \) contains the I = 2 type-1 polyphase components of the real prototype filter (6).

The equations (7) and (8) just differ by the additional rotation matrix \( D_{2}^{(p1)}(e^{-j\frac{2\pi}{4}}) \) and the sign of the argument of the polyphase matrix \( E_{2}^{(p1)}(z_{i}^{2}). \)

Exploiting the latter fact, the polyphase component block is shared by both FDMUX by introducing an additional sign alternation in the second FDMUX. The resulting novel efficient FDMUX block structure [1] is depicted in the left part of figure 3.

\[ \begin{bmatrix} \dddot{X}_{2}\left(z_{i}\right) \\ \dddot{X}_{2}\left(z_{i}\right) \\ \end{bmatrix} = \begin{bmatrix} \dddot{E}_{2}\left(z_{i}\right) \\ \dddot{E}_{2}\left(z_{i}\right) \\ \end{bmatrix}. \]

Figure 3: Efficient decomposed DFT polyphase SBC-FDMUX filter bank \((z_{0} = z_{i}^{2})\).
2.2 The SBC Filter Bank [1]

Figure 3 already contains the complement of the upper FDMUX to an SBC filter bank described by

\[
\hat{X}_{2}^{(ac)}(z_{1}) = \tilde{G}_2^{(ac)}(z_{2}) \bar{Y}_{2}(z_{2})^T \tilde{X}_{2}(z_{1}),
\]

where use is made of (3) and the \(M \times 1\) vector \(\hat{X}_{2}^{(ac)}(z_{1}) = [\hat{X}(z_{1}W_{2}^1)]_{k=0,1} \) contains the alias components of the reconstructed input signal. The \(I \times I\) matrix \(\tilde{G}_2^{(ac)}(z_{2}) = [G_{l=2c}(z_{2}W_{2}^k)]_{k=0,1}\) is the alias component matrix of the FDMUX filter bank in figure 1. In order to develop an efficient realisation, \(\tilde{G}_2^{(ac)}(z_{2})\) is decomposed similarly to (7):

\[
\tilde{G}_2^{(ac)}(z_{2}) = W_2D_2^{(p2)}(z_{2})R_2^{(p2,2)}(z_{2})W_{2},
\]

The diagonal type-2 \((p2)\) polyphase matrices \(D_2^{(p2)}(z_{2})\) and \(R_2^{(p2,2)}(z_{2})\) emerge from \(D_2^{(p1)}(z_{2})\) and \(R_2^{(p1,2)}(z_{2})\), respectively, by substituting \(p := I - 1 - p\) for \(p = 0, 1\) with \(I = 2\) in (6) [3, 5].

Perfect Reconstruction The SBC-FDFMUX filter bank is designed to meet the (N)PR conditions for a 2-channel SBC filter bank [3, 5]

\[
F_{d}(z_{1}) = \frac{1}{2} [G_{0}(z_{1})H_{0}(z_{1}) + G_{2}(z_{1})H_{2}(z_{1})] = z_{1}^{-k}
\]

(9)

\[
F_{a}(z_{1}) = \frac{1}{2} [G_{0}(z_{1})H_{0}(-z_{1}) + G_{2}(z_{1})H_{2}(-z_{1})] = 0,
\]

(10)

with \(k \in \mathbb{N}_0\) and where \(F_{d}(z_{1})\) is the distortion function and \(F_{a}(z_{1})\) the aliasing function [3, 5]. Hence, in case of PR, the output signal is given by

\[
\hat{X}(z_{1}) = z_{1}^{-k} \tilde{X}(z_{1}).
\]

(11)

3 SBC-FDFMUX Filter Bank Design

Each SBC-FDFMUX filter bank design is based on the design of a real prototype filter compliant with the SQMF or CQMF requirements [3, 5], respectively, from which all applied filters are derived by exploiting both the SQMF and CQMF relationships and the frequency shifting according to (1). In order to allow for a broad comparative evaluation of the suitability of these design methods for the given tasks and their computational efficiency, we adopt the filter specifications of [2] calling for a minimum stopband attenuation of \(a_{s} = 50dB\).

Due to the oversampling by two and in order to maintain the FDMUX capability as given in [2], the prototype transition band is chosen as wide as the passband, resulting in a passband edge at \(\Omega_{p} = \pi/4\) and a stopband edge at \(\Omega_{s} = 3\pi/4\).

3.1 Standard QMF Design [3, 5]

The SQMF approach derives all applied filters from one real linear-phase FIR prototype filter of odd order \(n_{SQMF} = N_{SQMF} - 1\). With SQMF, (10) is met exactly. Condition (9) calls for a power complementary prototype filter which, with linear-phase filters, can only be obtained approximately. Hence, in contrast to (11), SQMF has the NPR property.

Figure 4 depicts the approximately power complementary logarithmic magnitude response of the SQMF filters \(H_{P}(z_{1}) = H_{0}(z_{1})\) and \(H_{2}(z_{1})\) of length \(N_{SQMF} = 14\). These filters result from optimising the linear-phase prototype filter by merely adapting \(\Omega_{p}\) subject to an approximately minimum residual reconstruction error, e.g. the minimum deviation from the desired distortion function (9). The remaining residual deviation is shown in figure 5. For more sophisticated SQMF optimisation techniques refer to [4].

3.2 Conjugated QMF Design [3, 5]

The PR conditions (9) and (10) are exactly met by the CQMF approach. The real prototype filter \(H_{P}(z_{1}) = H_{0}(z_{1})\) is a spectral factor of a halfband filter \(HBF\) \(T(z_{1})\) with a non-negative zero-phase frequency response. The spectral factorisation allows for minimum or mixed phase prototype filters. Figure 6 shows the logarithmic (minimum phase) magnitude response of the CQMF prototype filter with \(N_{CQMF} = 14\) coefficients in comparison to the SQMF and the HBF FDMUX [2] prototype filters. The deviation of the CQMF design from the PR property, as to be seen from figure 5, is due to numerical errors of the MATLAB root finding procedure used for spectral factorisation.

4 COMPARISON

Table 1 comprises the design results for the SBC-FDFMUX filter bank. The achieved minimum stopband attenuations \(a_{s}\) are given in the first column. Moreover, \(N\) is the prototype filter length. The effective numbers of coefficients \(N'\) determines the computational load \(A_{CL}\), i.e. the number of multiplications per time unit. In case of the HBF FDMUX [2], \(N'\) is the number of non-zero coefficients, while for the SQMF filter banks \(N' = N/2\) due to coefficient symmetry. As for the CQMF filter banks, we have \(N' = N\). Since complex signals are downsampled by 2 and processed in real prototype filters, the computational load is given by \(A_{CL} = 2N' \cdot f_{i}/2\) [1].

The passband responses of all FDMUX, i.e. the prototype filters, are shown in figure 7. The corresponding peak-to-peak deviations \(\Delta a_{pp}\) are listed
in Table 1, where $\Delta a_{pp}$ denote the residual reconstruction errors of the SBC filter banks.

<table>
<thead>
<tr>
<th>Design Approach</th>
<th>$a_s$</th>
<th>$N$</th>
<th>$N'$</th>
<th>$A_{CL}$</th>
<th>$\Delta a_{pp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HBF-FDMUX [2]</td>
<td>56</td>
<td>11</td>
<td>7</td>
<td>$7f_i$</td>
<td>0.0275</td>
</tr>
<tr>
<td>HBF-SBC</td>
<td>infeasible</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SQMF-FDMUX</td>
<td>51</td>
<td>14</td>
<td>7</td>
<td>$7f_i$</td>
<td>0.0470</td>
</tr>
<tr>
<td>NPR-SQMF-SBC</td>
<td></td>
<td></td>
<td></td>
<td>14$f_i$</td>
<td>0.2551</td>
</tr>
<tr>
<td>CQMF-FDMUX</td>
<td>57</td>
<td>14</td>
<td>14</td>
<td>$14f_i$</td>
<td>1.6 · 10^{-5}</td>
</tr>
<tr>
<td>PR-CQMF-SBC</td>
<td></td>
<td></td>
<td></td>
<td>28$f_i$</td>
<td>(1.6 · 10^{-5})</td>
</tr>
</tbody>
</table>

Table 1: Comparison of results ($a_s$, $\Delta a_{pp}$ in dB).

Even though the $A_{CL}$ of the HBF- and the SQMF-FDMUX are comparable, only the latter can be used as an SBC subsystem. Comparing the efficiency of the PR CQMF and the NPR SQMF SBC, the latter outperforms the former twice. In addition to its good overall performance, the SQMF-FDMUX has the highly desirable linear-phase property [2].

5 CONCLUSION

In this paper we investigate and compare two design options of the SBC-FDMUX filter bank. The NPR SQMF approach clearly outperforms the PR CQMF design approach in terms of computation and because of its linear-phase FDMUX subsystem, whereas the relative advantages of the CQMF approach (PR property and smaller FDMUX pass-band deviations, cf. figures 5 and 7) are marginal. Lacking power complementarity, the efficient reference HBF-FDMUX [2] cannot be used as FDMUX subsystem of an SBC-FDFMUX filter bank.

References