Flexible Bandwidth Reallocation MDFT SBC-FDFMUX Filter Bank for Future Bent-Pipe FDM Satellite Systems

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INTRODUCTION

Future bent-pipe FDM satellite systems [1,2,3,4] ask for a high flexibility concerning bandwidth-to-user assignment, which is achieved by merging several elementary channels (granules) to one wideband channel, as required by the users. For instance, a respective scenario is described in [5,6,7], where the requirements are most challenging for the forward uplink: the overall capacity of 40GHz is realised by 10 gateway beams of 2x2GHz using dual polarisation in the uplink. After separation and down-conversion of the two polarisation components to a suitably low intermediate frequency, each 2GHz signal is split into two signals of a bandwidth of about 1GHz applying analogue filtering in order to alleviate the A-to-D interface (ADI) requirements. High speed ADI digitise these 1GHz signals at a sampling frequency of \( f = 2.4 \text{GHz} \).

In the digital MIMO processor systems the incoming broadband FDM signals (uplink beams) are always channelised to granularity level, i.e. decomposed into sub-signals of identical bandwidth, using parallelly arranged frequency demultiplexers (FDMUX). On-board switching is applied to allow for flexible adjustment of channel-to-beam allocation, i.e. any granule can potentially be switched to any of the parallelly arranged frequency multiplexers (FMUX), which provide the remultiplexed processor output signals (forward downlink beams).

For the channelisation of each digitised FDM signal, three different cases must be distinguished: i) All granules represent signals (carriers) of independent users, ii) all granules belong to a single wideband carrier of a specific user and iii) as a combination of the first two cases, the FDM input signal is composed of granular (case i) and wideband signals of bandwidths ranging from two to ten granules (case ii) [5,6,7]. A subsequent on-board switch feeds up to 200 FDM remultiplexers (FMUX) with the assigned granules to form downlink user beams of an overall bandwidth of about 200KHz each. As for case i, the FDMUX-FMUX cascade (FDMUX filter bank) merely recombines independent granules. Case ii requires a sub-band coder (SBC) filter bank that (nearly) perfectly reconstructs ([N]-PR) the granules of the original wideband signal [8,9]. Case iii calls for a combined [N]-PR SBC-FDFMUX filter bank [7,10,11,12].

Efficient oversampling tree-structured SBC-FDFMUX approaches have been proposed by Abdulazim and Göckler [7,10,11,12]. However, due to its underlying tree-structure, this SBC-FDFMUX filter bank approach is still restricted to power-of-two channel numbers. Furthermore, only channel numbers of powers of two can be merged to form one single wideband channel. Finally, potential on-board switching is restricted, i.e. only centre frequencies that fit into the pattern of the tree-structured SBC-FDFMUX filter bank can be realised for the reallocated wideband signals. Independently of [7,10,11,12], Johansson and Löwenborg presented a flexible frequency-band reallocation network based on oversampled complex-modulated filter banks [13] overcoming these limitations.

This paper presents a novel filter bank approach for flexible bandwidth reallocation based on maximally-decimating N-PR Modified DFT (MDFT) filter banks [14,15,16]. The main feature of the MDFT filter bank is its robust, structurally imposed aliasing compensation that allows for applying critical sampling, which simplifies the filter bank design procedure compared to [13]. First of all, the feasibility of MDFT filter banks for the desired filter bank tasks, as described above, is investigated. In contrast to [16], modifications have to be introduced in order to easily combine the structurally imposed aliasing compensation and the desired switching functionality within the novel N-PR MDFT SBC-FDFMUX filter bank, which represents a clear and straightforward approach for arbitrary filter bank channel numbers with the desired full flexibility.

For a better understanding, we would like to note that the parameter \( K \) denotes the number of filter bank channels. In case of critical sampling \( K \) equals the decimation factor \( M \). The parameter \( Q \) denotes the number of elementary granules.
an FDM signal comprises. Hence, the parameter $Q$ is generally different from $K$. Furthermore, complex-valued signals and $z$-Transforms thereof are underlined.

**THE MODIFIED DFT SBC FILTERBANK**

**Channel Filter Functions**

MDFT SBC filter banks (cf. Fig.1) belong to the family of complex modulated DFT SBC filter banks [8,9], where a real prototype lowpass filter $h_p(n)\rightleftharpoons H_p(z)$ is decomposed in its polyphase components, and all channel filter functions are obtained by a uniform complex modulation that is efficiently realised by the IFFT/FFT algorithm [8,9]. Hence, with $k = 0, \ldots, K-1$ and $W^k_k = e^{j2\pi k}$ we get

$$H_k(z) = H_p(z)W^k_k$$

(1)

$$G_k(z) = MH_p(z)W^k_k$$

(2)

where $H_k(z)$ and $G_k(z)$ represent the channel filters of the FDMUX (FMUX) filter bank. The integer parameters $n_H$ and $n_G$ define the initial phases of the complex modulation of the analysis and synthesis filters, respectively. For linear phase FIR filters, as assumed throughout this paper, these two parameters are determined by [16]

$$n_H = n_0/2 + \alpha M/4$$

(3)

$$n_G = n_0 - n_H$$

(4)

with the arbitrary number $\alpha \in \mathbb{Z}$ and $n_0 = d_0 \text{ modulo } M = ((d_0)_M)$, where $d_0 = N-1$ is the overall filter bank delay and $N$ the prototype filter length [16].

![Fig.1](image)

Fig.1. The maximally decimating $K$-channel MDFT SBC filter bank ($M=K$) in parallel structure (first three channels).

**Prototype Filter**

The prototype filter of a critically sampling N-PR MDFT SBC filter bank (channel number $K$ = decimation factor $M$) is designed in such a way that the complex-modulated channel filter functions in (1) and (2), respectively, meet the Pseudo-QMF (PQMF) conditions [17,18]: i) The transfer functions of any two contiguous channels are approximately power complementary in the range between their centre frequencies, and hence, the overall distortion function of the filter bank is sufficiently close to a mere delay function. ii) Furthermore, the main aliasing spectra directly neighbouring the useful spectra are compensated by proper methods (cf. Fig.1). iii) Finally, all other aliasing spectra are eliminated by a sufficiently high stopband attenuation of the prototype filter.

The first and third requirements specify the design of the prototype filter. In the scope of this paper, we apply the fast and efficient frequency-sampling filter design method proposed in [19], which yields a linear-phase lowpass prototype filter for N-PR. The second PQMF requirement calls for structural modifications [14]. Only these structural modifications allow for the application of critical sampling.

**Structural Modification for Alias Compensation**

The critically sampling $K$-channel MDFT SBC filter bank, which compensates the main aliasing spectra contiguous to the useful spectra, is derived from the oversampling $K$-channel DFT SBC filter bank with a decimation factor of
\( M' = M/2 = K/2 \) by the following modifications \[14\] (cf. Fig.1): i) Each subband signal is decomposed into two polyphase components, resulting in a two step decimation with a total decimation factor of \( M \). ii) Subsequently, solely the imaginary and the real parts of each subband signal are processed alternatingly, according to Fig.1. iii) Before reconstruction, the two polyphase components of each subband signal are recomposed.

References \[16,15\] describe the impact of this modifications on alias compensation as mentioned above. Furthermore, it is shown that the structure of the MDFT filter bank allows for the cancellation not only of the neighbouring main alias spectra, but also of further alias spectra which are, anyway, sufficiently suppressed by the stopband attenuation of the prototype filter.

In the following, we describe the structurally imposed aliasing compensation for \( K = M = 2 \), exclusively, in order to illuminate the general principles of the modification, and as a reference for the following investigations. The parallel structure of a 2-channel MDFT SBC filter bank with its structurally imposed aliasing compensation is depicted in Fig.2. It should be noted that even though the MDFT filter bank is now considerably simplified, its principles can yet be explained herewith.

In the following, the \( z \)-Transforms of the complex-valued input signal \[15\]
\[
X(n) = x^{(R)}(n) + j \cdot x^{(I)}(n)
\] (5)
and its complex-conjugate are given by
\[
\overline{X}(z_i) = X^{(R)}(z_i) + X^{(I)}(z_i)
\] (6)
\[
\overline{X}(z_i) = X^{(R)}(z_i) - X^{(I)}(z_i),
\] (7)
where
\[
x^{(R)}(n) \leftrightarrow z^T \rightarrow X^{(R)}(z_i)
\] (8)
\[
j \cdot x^{(I)}(n) \leftrightarrow z^T \rightarrow X^{(I)}(z_i).
\] (9)

For the explanation of the structurally imposed aliasing compensation, the real and imaginary part blocks in Fig.2 are shifted in front of the downsamplers \[8, 9\]. Hence, the real parts and the imaginary parts multiplied by \( j \) of the subband signals are given by
\[
\begin{align*}
Z \left[ \text{Re} \left\{ \overline{X}(n) \right\} \right] &= Z \left[ \text{Re} \left\{ h_n(n) * x(k) \right\} \right] = \frac{H_k(z_i)}{2} \left[ X(z_i) + \overline{X}(z_i) \right] \\
Z \left[ j \text{Im} \left\{ \overline{X}(n) \right\} \right] &= Z \left[ j \text{Im} \left\{ h_n(n) * x(k) \right\} \right] = \frac{H_k(z_i)}{2} \left[ X(z_i) - \overline{X}(z_i) \right].
\end{align*}
\] (10) (11)

As a result of \( K = M = 2 \), \( h_n(n) \) (with \( k = 0,1 \)) are real-valued. Due to the exclusive processing of real part and the imaginary part times \( j \) of the respective subband signals, the respective original spectral component represented by \( H_k(z_i) \cdot X(z_i) \) is overlapped by the respective mirror spectra, viz. \( + H_k(z_i) \cdot \overline{X}(z_i) \) and \( - H_k(z_i) \cdot \overline{X}(z_i) \). This indicates the potential for aliasing compensation.

The downsampled signals of the upper branch of the filter bank result in \[8, 9\]
\[ Y_0^{(R)}(z_i) = \frac{1}{4} \sum_{n=0}^{\frac{K}{2}-1} H_0(z_iW_n) \left[ X(z_iW_n) + \overline{X}(z_iW_n) \right] \]  
\[ Y_0^{(l)}(z_i) = \frac{z_i^{-1}}{4} \sum_{n=0}^{\frac{K}{2}-1} H_0(z_iW_n) \left[ X(z_iW_n) - \overline{X}(z_iW_n) \right] W_n \]  
\[ Y_1^{(R)}(z_i) = \frac{1}{4} \sum_{n=0}^{\frac{K}{2}-1} H_1(z_iW_n) \left[ X(z_iW_n) + \overline{X}(z_iW_n) \right] \]  
\[ Y_1^{(l)}(z_i) = \frac{z_i^{-1}}{4} \sum_{n=0}^{\frac{K}{2}-1} H_1(z_iW_n) \left[ X(z_iW_n) - \overline{X}(z_iW_n) \right] W_n \]

where \( z_i = z_i^0 \). Correspondingly, the downsampled signals of the lower branch are

\[ Y_0^{(R)}(z_i) = \frac{1}{4} \sum_{n=0}^{\frac{K}{2}-1} H_0(z_iW_n) \left[ X(z_iW_n) - \overline{X}(z_iW_n) \right] \]  
\[ Y_0^{(l)}(z_i) = \frac{z_i^{-1}}{4} \sum_{n=0}^{\frac{K}{2}-1} H_0(z_iW_n) \left[ X(z_iW_n) + \overline{X}(z_iW_n) \right] W_n \]  
\[ Y_1^{(R)}(z_i) = \frac{1}{4} \sum_{n=0}^{\frac{K}{2}-1} H_1(z_iW_n) \left[ X(z_iW_n) + \overline{X}(z_iW_n) \right] \]  
\[ Y_1^{(l)}(z_i) = \frac{z_i^{-1}}{4} \sum_{n=0}^{\frac{K}{2}-1} H_1(z_iW_n) \left[ X(z_iW_n) - \overline{X}(z_iW_n) \right] W_n \]

For instance, the signal \( Y_0^{(R)}(z_i) \) consists of four components: first, of the filtered original spectrum \( H_0(z_i) \cdot X(z_i) \) and its mirror spectrum \( H_0(z_i) \cdot \overline{X}(z_i) \), and second, of their first modulation components \( H_0(z_iW_n) \cdot X(z_iW_n) \) and \( H_0(z_iW_n) \cdot \overline{X}(z_iW_n) \). The signal \( Y_0^{(l)}(z_i) \) comprises, besides a simple delay, the same components. However, due to the processing of the imaginary part multiplied by \( j \) and the phase-shifted downsampling, here \( H_0(z_i) \cdot \overline{X}(z_i) \) as well as \( H_1(z_iW_n) \cdot \overline{X}(z_iW_n) \) have opposite sign. Correspondingly, this is also principally true for the signals \( Y_1^{(l)}(z_i) \) and \( Y_1^{(R)}(z_i) \). Hence, after upsampling the subband signals and their addition, as shown in Fig.2, we get

\[ \hat{X}_0(z_i) = z_i^{-1} Y_0^{(R)}(z_i) + Y_0^{(l)}(z_i) \]  
\[ \hat{X}_1(z_i) = z_i^{-1} Y_1^{(l)}(z_i) + Y_1^{(R)}(z_i) \]

where the components with different signs are compensated, as described above. Finally, the filter bank output signal results in

\[ \hat{X}(z_i) = \frac{z_i^{-1}}{2} \left( G_0(z_i) \left[ H_0(z_i) \overline{X}(z_i) + H_0(z_iW_n) \overline{X}(z_iW_n) \right] + G_1(z_i) \left[ H_1(z_i) \overline{X}(z_i) - H_1(z_iW_n) \overline{X}(z_iW_n) \right] \right) \]
\[ = \frac{z_i^{-1}}{2} \left( [G_0(z_i) H_0(z_i) + G_i(z_i) H_i(z_i)] \overline{X}(z_i) + [G_0(z_i) H_0(z_iW_n) - G_i(z_i) H_i(z_iW_n)] \overline{X}(z_iW_n) \right) \]

With

\[ G_0(z_i) = 2H_0(z_iW_n) \]  
\[ G_i(z_i) = 2H_i(z_iW_n) \]

due to complex modulation (cf. (1) and (2)), we finally get

\[ \hat{X}(z_i) = \frac{z_i^{-1}}{2} \sum_{k=0}^{d-1} G_k(z_i) H_k(z_i) \overline{X}(z_i) \]

Equation (21) shows that the remaining aliasing components are fully compensated by the FMUX. In case of \( \sum_{k=0}^{d-1} G_k(z_i) H_k(z_i) \overline{X}(z_i) = c \cdot z_i^{-d} \) with \( c = \text{const.} \) and \( d \in \mathbb{N} \), we have a PR filter bank with power-complementary filters. If (21) is nearly met, we have an N-PR filter bank with nearly power-complementary filters.

For the general description of aliasing compensation for \( K = M > 2 \) and efficient implementations of the MDFT SBC filter bank refer to [16].

**THE MDFT SBC-FDFMUX FILTER BANK**

For the investigation of the feasibility of MDFT filter banks in connection with switching functions for flexible bandwidth reallocation, we again start the following considerations with \( K = M = 2 \). Subsequently, we generalise the achieved results to higher channel numbers.
On-Board Switching for $K = M = Q = 2$

Fig.3 depicts the 2-channel MDFT filter bank with a switching function. Here, in comparison to Fig.2, the FDMUX output signals are switched to different inputs of the FMUX.

![Diagram of 2-channel MDFT SBC filter bank with switching function](image)

The maximally decimating 2-channel MDFT SBC filter bank in parallel structure with switching function.

Following (18), this is now described by

$$\hat{X}(z_i) = \frac{z_i^{-1}}{2} \left( G_0(z_i) \left[ H_0(z_i) X(z_i) - H_1(z_i, W_2) \overline{X}(z, W_2) \right] + G_1(z_i) \left[ H_0(z_i) X(z_i) + H_1(z_i, W_2) \overline{X}(z, W_2) \right] \right).$$  \hfill (22)

The components $H_1(z_i) X(z_i)$ and $H_0(z_i) \overline{X}(z)$, which, according to (17), are required to (nearly) perfectly reconstruct the input signal, are now allocated at the stopband region of $G_0(z_i)$ and $G_1(z_i)$, respectively. Hence, an additional frequency shift by $W_2^* = (-1)^* \frac{2}{2}$ directly after the upsampling by two is needed to reallocate these components. It is introduced as shown in Fig.3. Hence, we get

$$\hat{X}(z_i) = \frac{z_i^{-1}}{2} \left( G_0(z_i) \left[ H_0(z_i, W_2) \overline{X}(z, W_2) - H_1(z_i) \overline{X}(z) \right] + G_1(z_i) \left[ H_0(z_i, W_2, \overline{X}(z), W_2) + H_1(z_i) \overline{X}(z) \right] \right),$$  \hfill (23)

exploiting $W_2^2 = 1$. By applying these frequency shifts, the structurally imposed aliasing compensation is fully retained, and 2-channel MDFT filter banks are applicable for flexible bandwidth reallocation. As such, they can readily be used in tree-structured SBC-FDFMUX filter banks [7,10,11,12].

On-Board Switching for $K = M = Q > 2$

For $K = M > 2$, the switching function has, in principle, the same impact on the spectral components, as described above for $K = M = 2$. The aliasing compensation requires that the original spectral components must be allocated at the passband region of the respective synthesis filter of the FMUX. In connection with a switching function this is not given, if the elementary granule is shifted over an odd number of filter bank channels, as shown above for the case of $K = M = 2$. Hence, in this case, a frequency shift by $W_2^* = (-1)^* \frac{2}{2}$ is needed for spectral reversion. On the other hand, if an elementary granule is shifted over an even number of filter bank channels, a frequency shift must not be applied.

On-Board Switching for $M = K = 2Q > 2$

The results of the previous section motivates the splitting of each elementary granule into two sub-granules leading to a filter bank with $K = M = 2Q$ channels, i.e. with twice the number of elementary granules. By this approach, each sub-granule may exclusively be switched over an even number of filter bank channels. Hence, the application of frequency shifts for retaining the structurally imposed aliasing compensation is no longer required in any form. Each sub-granule is (nearly) perfectly reconstructed corresponding to wideband signals.

A second advantage becomes obvious from Fig.1: Since downsampling (upsampling) by $M / 2$ is applied, the overall downsampling factor $M$ and, hence, the number of filter bank channels $K$ must be even. As a consequence, by setting $K = M = 2Q$, the MDFT SBC-FDFMUX filter bank can be designed for FDM-signals, which are composed of an odd number $Q$ of elementary granules.
DESIGN EXAMPLES

For all following examples, as shown in Figs.4 and 8, we consider an FDM input spectrum of $Q = 8$ elementary granules. It comprises a first two-band signal (“$A$”) (two merged elementary granules), a three-band signal (“$B$”), a second two-band signal (“$C$”) and a single band signal (“$D$”), as depicted in Fig.4 from left to right according to Fig.4. These signal spectra are switches to the positions as shown in Fig. 7: “$B$” -“$D$” -“$C$” -“$A$”.

Example for $Q=K=M=8$

The filter bank of this first example is designed for $Q = K = M = 8$, where the number of filter bank channels equals the number of elementary granules. The sampling frequency of the input signal is $f_s = Q \cdot 12\text{MHz} = 96\text{MHz}$. The real prototype filter has a passband edge at $f_p = 5\text{MHz}$ and a stopband edge at $f_s = 7\text{MHz}$. Its length is set to $N = 383$ to achieve a stopband rejection of better than 60dB. The logarithmic filter frequency responses of the FDMUX are depicted in Fig. 5.

The output spectrum after switching is shown in Fig.6, where no additional frequency shift (sign alternation) is performed. Hence, the signal “$C$” is not perfectly reconstructed, since its elementary granules are shifted over one filter bank channel. All other elementary granules are shifted over an even number of filter bank channels, and hence, are perfectly reconstructed. Finally, Fig.7 depicts the output spectrum, where the necessary frequency shifts are applied. The signal “$C$” is now also perfectly reconstructed.

Example for $2Q=K=M=16$

In practice, all input signals are generally oversampled by a factor of two for efficiency reasons [12]. Therefore, we consider a second example with $2Q = K = M = 16$. Here, only half of the overall bandwidth of the input signal is loaded, as shown in Fig.8. The input sampling frequency is now $f_s = 2 \cdot 12\text{MHz} = 192\text{MHz}$. With a passband edge at $f_p = 5\text{MHz}$ and a stopband edge at $f_s = 7\text{MHz}$ the prototype filter length is set to $N = 767$ to get the same performance as in the previous example. The resulting logarithmic filter frequency responses of the FDMUX are depicted in Fig. 9. The achieved results in Figs.10 and 11 can directly be compared with those of the first example (Figs.6 and 7).
Example for $4Q=K=M=32$

For this third and last example with the oversampled input signal according to Fig. 8 the application of additional frequency shifts is not necessary, since each elementary granule is split into two sub-granules (see above). The real prototype filter has a passband edge at $f_p = 2\text{MHz}$ and a stopband edge at $f_s = 4\text{MHz}$. The filter length is now set to $N = 1279$ for corresponding performance. Fig.12 shows the logarithmic filter frequency responses of the FDMUX, and Fig.13 the reallocated output spectrum.

**CONCLUSION**

In this paper we present SBC-FDFMUX filter banks for flexible bandwidth re-allocation that are based on Modified DFT filter banks. At first, we recall the principal of the structurally imposed aliasing compensation of MDFT filter banks that allows for applying critical sampling. Subsequently, the applicability of this type of filter banks for flexible bandwidth reallocation is investigated. By introducing additional frequency shifts (sign alternations) in conjunction with
channel switching, this aliasing compensation is readily retained. It is shown that these frequency shifts are not necessary, if the filter bank channel number is twice the number of elementary granules and each granule is decomposed into two sub-granules, respectively.

A main advantage of MDFT SBC-FDFMUX filter banks compared to tree-structured SBC-FDFMUX filter banks [7,10,11,12] is their higher flexibility allowing for any potential channel centre frequency and any potential number of merged granules.

In future research, we will investigate more efficient implementations of the MDFT SBC-FDFMUX filter bank by applying polyphase decomposition of the prototype filter. This will allow for fair performance comparison of this highly flexible filter bank approach as to channel reallocation (switching) with other proposals [7,10,11,12,13]. In addition, prototype filter designs will be revisited to reduce the necessary filter order and, hence, the overall expenditure. Finally, suitable frequency offsets of the transfer functions of the filter bank are introduced as required.

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